Calculations
for an
Implementation of the Absorption Cooled Energy Tower

Robert J. Rohatensky
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Jupiter Beach FLA, USA

Tropical Climate, High Humidity, 12 months of > +20C temperatures

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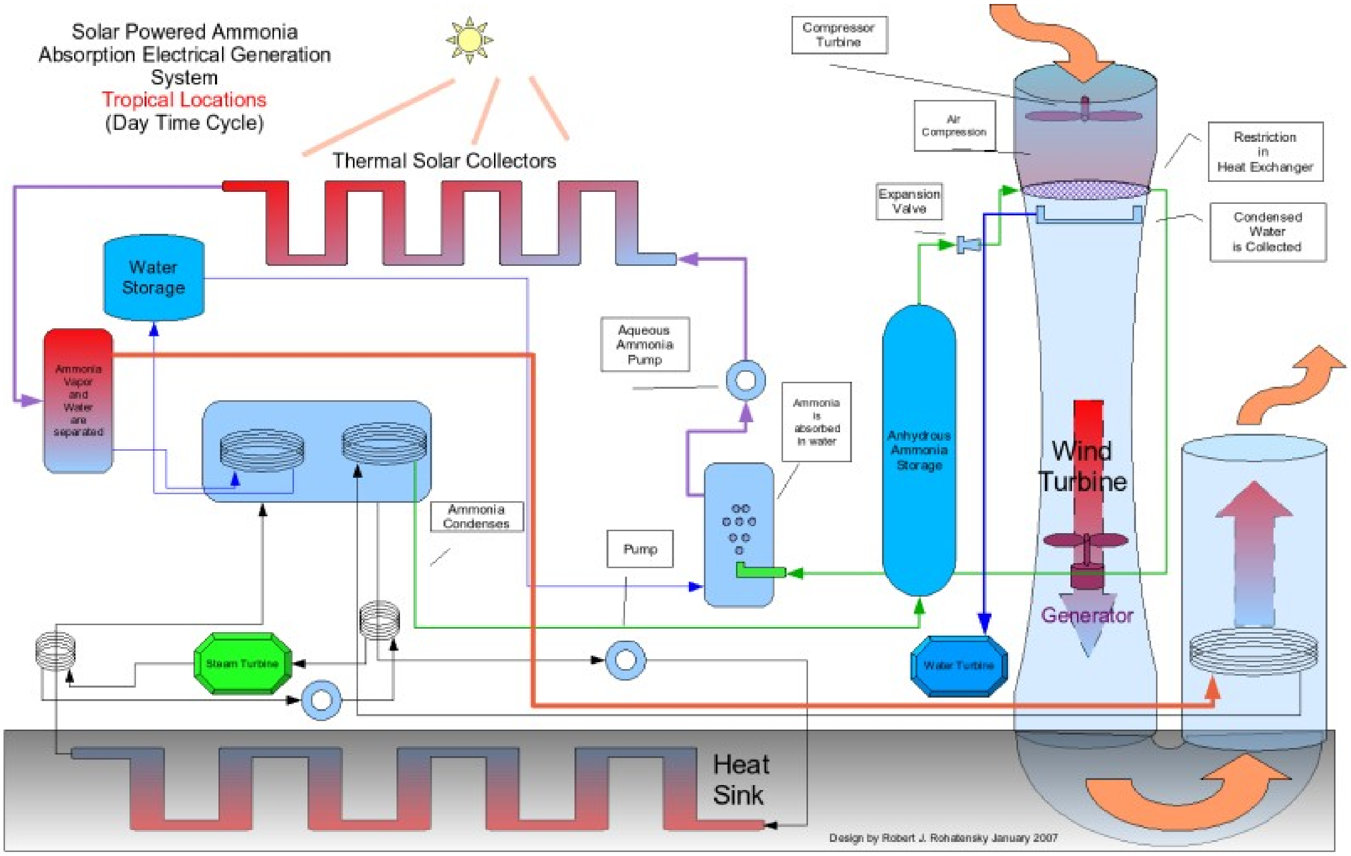
Overview

This location with warm temperatures, high solar isolation and humidity would best be suited by a system that continually functions in a down draft mode with added air compression to extract as much water as possible from the air at the top of the tower. A compression turbine and restriction through the cooling heat exchanger would increase the air pressure and temperature through the cooling coils to improve thermal transfer. The air would decompress below the cooling coils with most of the water vapor extracted and the cool dry air would have a high negative buoyancy.

The fact that water vapor is lighter than air is counter-intuitive for many people. The density of dry air at sea level is 1.293 g/L and water vapor is 0.804 g/L. Extracting the water vapor from the air and cooling it causes the air to be much more dense than the ambient air outside the tower.

The very high absolute humidity in this location creates a large amount of condensed water capture. This water is substantial and besides being potable or usable for hydrogen electrolysis can be used for additional power generation.

High Humidity Tropical System



Night Operation

This system would continually operate in down-draft mode cooling the air. This would require substantial anhydrous and aqueous ammonia storage and large thermal solar collectors to recover all of the ammonia used during sunlight periods.

Climate

NASA Surface meteorology and Solar Energy

At Latitude 50 and Longitude -104
Average elevation: 584 meters

Monthly Averaged Insolation Incident On A Horizontal Surface (kWh/m2/day)

Lat 26.93 Lon -80.17	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual Average
10-year Average	3.28	4.11	4.87	5.88	5.99	5.48	5.32	5.18	4.63	4.21	3.44	3.06	4.62

Monthly Averaged Air Temperature At 10 m Above The Surface Of The Earth (° C)

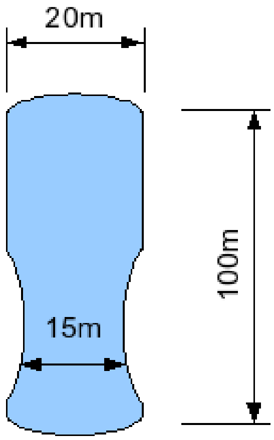
Lat 26.93 Lon -80.17	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual Average
10-year Average	19.9	20	21.1	22.4	24.9	26.8	27.3	27.5	26.8	25	23	20.6	23.8

Monthly Averaged Relative Humidity (%)

Lat 26.93 Lon -80.17	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
10-year Average	71.4	69.2	68.4	65.7	69.3	71.9	71	71.1	73.3	72.6	73.5	72.1

Tower Design

The power output of the wind turbine in the tower is based on the air velocity and the diameter of the tower. The air velocity is based on the buoyancy of the air external to the tower, the drag loss across the heat exchanger and tower walls and the exit loss. In this location the annual high humidity and temperature lends itself to a focus on lowering both the temperature and the water vapor percentage inside the tower. These two factors affect the negative buoyancy of the air in the tower. Because there is no energy expended in expanding the pressurized ammonia in the cooling coils, the power output of the system is based on the height and diameter of the tower, the surface area of the heat exchanger and the ambient air temperature and humidity.

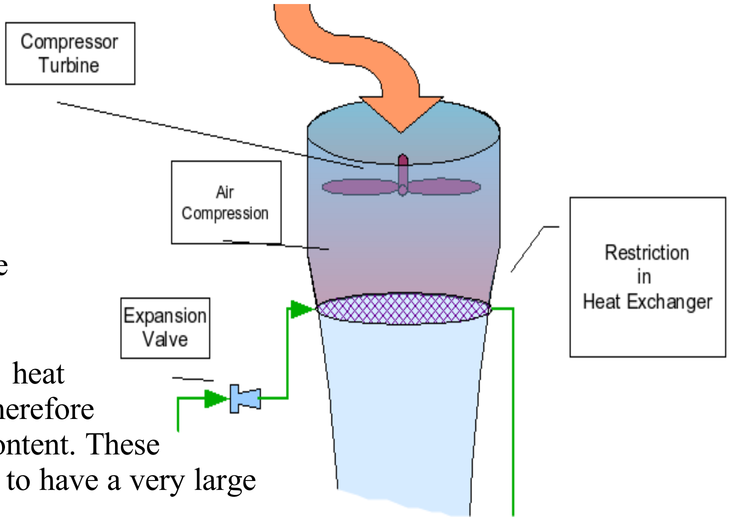


Assuming a medium scale system with a tower height of 100m and a diameter of 20m narrowing to 15m in the lower section. The tower will function in a continual down-draft mode.

A compressor turbine is added to the top of the tower to increase the air pressure above the heat exchanger. This increase in air pressure increases the temperature of the air and allows for a larger temperature difference between the intake air and the heat exchanger plates and also increases the contact factor.

The cold plates of the heat exchanger are at -33C and as the air is forced through under pressure, almost all of the moisture in the air will condense on the heat exchanger.

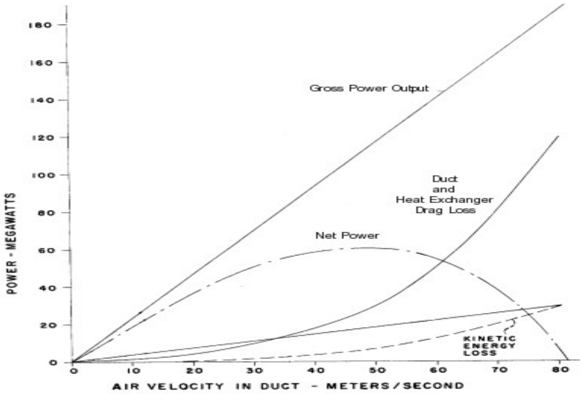
As the air leaves the restriction of the heat exchanger, it will drop in pressure (therefore temperature) and have a very low moisture content. These factors will cause the internal air in the tower to have a very large negative buoyancy factor.



The high humidity means that a very large amount of water will condense on the heat exchanger and this distilled water by-product may be utilized domestically, for irrigation or for hydrogen electrolysis.

Wind Turbine Power Calculation (Summer Months)

This chart from patent US003894393 for the water spray tower has a peak net output with a wind velocity of 50 m/s. The ambient air temperature and tower height are too low to achieve this air velocity, but a sustained 20 m/s (72 km/h) is a reasonable target air velocity. The system will be designed to control the ammonia flow and the resulting air cooling to keep the air velocity in the tower at a constant rate. A constant air flow will allow the system to have a reliable turbine rotation speed and to utilize AC alternators at a fixed frequency rather than DC systems and inverters for AC output. This lowers the cost of the generating system substantially over traditional wind turbines with varying RPM and DC generators.



Target Air Velocity: 20m/s

Area of turbine: $A=\pi r^2$

$A=7.5^2\times 3.14159$

$176.71\,m^2 - \text{hub area} = 150\,m^2$

Air volume through turbine: $V_a=U\times A$

$20\,m/s\times 150\,m^2=3000\,m^3/s$

To achieve the desired 20 m/s air velocity 3000 cubic meters of air per second need to be moving through the turbine. The top and bottom of the tower are wider so although the same volume of air is moving into the top of the tower the air velocity is lower through the larger area. The exit loss where the output air meets the external air is reduced by having a larger area and lower velocity.

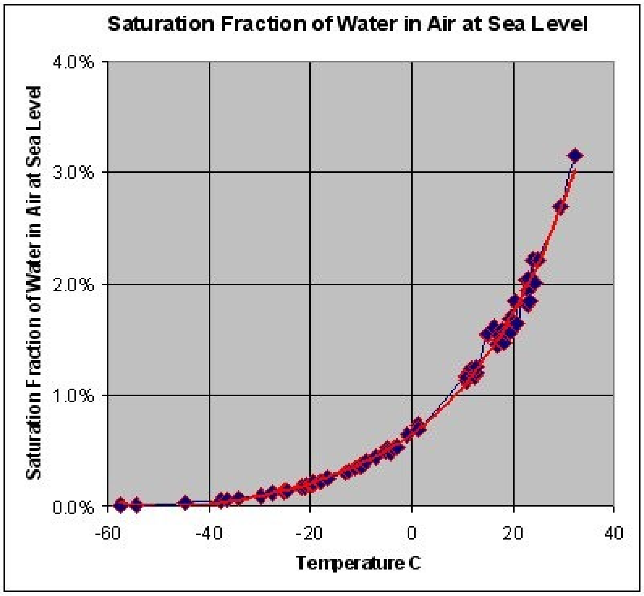
Area of top inlet: $A=\pi r^2$

$A=3.14159\times 10^2$

$314.59\,m^2 - \text{heat exchanger area} = 200\,m^2$

$V_a=U\times A$

$3000\,m^3/s \div 200\,m^2 = 15\,m/s$



This assumes a heat exchanger horizontal surface area of ~ 115 m^2.

The goal is to remove the required amount of heat and water from the air at the top of the tower to cause the negative buoyancy to achieve the required air velocity across the tower height. In this model the condensed air is allowed to drip off the cooling coils and is ignored in the power calculation.

The formula for air density is:

$$\rho=\frac{p}{R\times T}$$

The specific gas constant "R" for dry air is:

$$R_{dry,air}=287.05\frac{J}{kg\times K}$$

The target is to lower the 24°C average ambient air to 0°C.
For dry air the density at 101.00 kPa at 24°C:

$$\frac{10100/287.05}{297.15}=1.184\,kg/m^3$$

For 24°C air at 70% relative humidity, the air is approximately 2.1% water vapor with a density of 0.804 kg/m^3. This offsets the density by 0.804*.021:

$$1.184 \times 0.979 \\ +0.804 * 0.021 \\ =1.176\,kg/m^3$$

For dry air the density at 101.00 k Pa at 0°C:

$$\frac{10100/287.05}{273.15} = 1.288\,kg/m^3$$

Although the relative humidity of the air in the tower after cooling is 100% (below outside air dew point) the 0°C temperature means that the absolute humidity is at approximately 0.6 %.

$$1.288 \times 0.994 \\ +0.804 \times 0.006 \\ =1.285\,kg/m^3$$

	External	**Internal**
External Air Temperature	24°C (297.15°K)	0°C (273.15°K)
Relative Humidity	70%	100%
Air Pressure (586m + 100m above sea level)	101.00 k Pa	101.00 k Pa
Air Density ρ	1.285 kg/m³	1.176 kg/m³

The volume of the air inside the tower is 2 truncated cones:

$$V = \Pi (R^2 + rR + r^2) h/3 + \Pi (R^2 + rR + r^2) h/3 \\ V = \Pi \times (10^2 + 10 \times 7.5 + 7.5^2) \times 75 + \Pi \times (10^2 + 10 \times 7.5 + 7.5^2) \times 25 \\ V = 72649.33\,m^3$$

The available potential energy is the negative buoyancy of the denser dry cold air inside the tower relative to the air outside the tower.

$$F = V \times (\rho_i - \rho_o) \times 9.8 \\ F = 72649.33 \times 1.285 - 72649.33 \times 1.176 \times 9.8 \\ F = 77603.01\,N$$

The average velocity of the air moving down the tower without drag:

$$v_a = \frac{\sqrt{2gd}}{2}$$
$$V_a = \frac{\sqrt{2 \times 9.8\, m/s^2 \times 100\, m}}{2}$$
$$V_a = 22.14\, m/s$$

The peak velocity of the air after it falls 100m (excluding exit loss):

$$v_a = \sqrt{2gd}$$
$$V_a = \sqrt{2 \times 9.8\, m/s^2 \times 100\, m}$$
$$V_a = 44.27\, m/s$$

The assumption is that with exit loss the air moving through the turbine would be near the average velocity. The exit loss is substantial and is caused by the downdraft air having to push the static air at the bottom. With exit loss, the real velocity of the air will be much less than the peak velocity.

The design of this particular system includes a compression turbine. The compression turbine and it's affect on the air velocity is not taken into account in the power calculation, but it's power input will be subtracted. The increase in velocity due to the compression turbine shouldn't affect the average or peak velocity down the 100m tower substantially.

The gross power using a hydro formula is the flow rate in kg/s (of buoyancy) times 9.8 times height.

$$W = flow\ rate \cdot g \cdot h$$
$$W = (1.285 - 1.176) \times 3000 \times 9.8 \times 100$$
$$W = 199920 = 320\, kW\ gross\ power$$

Using a wind turbine formula and the 22.14 average wind speed with a 0.33 turbine power efficiency the actual power is:

Power delivered = Cp x area of wind turbine x $^1/_2\, \rho v^3$

$$P = 0.33 \times 150 \times \frac{1}{2} \times 1.285 \times 22.14^3$$
$$P = 345\, kW$$

The efficiency of the wind turbine is much better than a conventional natural wind turbine due to much lower tip loss because the turbine in a duct. The assumption is that with a well designed turbine a much larger portion of the energy could be converted to electricity than in a free standing wind turbine. The air in the tower can also be routed into a vortex to increase the angle of attack and efficiency.

The estimated electrical power output of the wind turbine in the tower is ~ 350kW consistently 24 hours per day.

Alternate Flow Calculations Using the Stack Effect Formula

Another method of calculating the air flow through the tower is by using the Stack Effect formula:

$$Q = C A \sqrt{2 g h \frac{T_h - T_c}{T_h}}$$

Q = stack effect flow rate, m^3/s
A = flow area, m^2
C = discharge coefficient (usually taken to be from 0.65 to 0.70)
g = gravitational acceleration, 9.8 m/s^2
h = height, m
T_h = warm temperature, K
T_c = cold temperature, K

This will give the flow rate in m^3/s, but doesn't take into account differences in absolute humidity.

$$0.65 * 314 * \sqrt{2 * 9.8 * 100 * \frac{(297.15 - 273.15)}{297.15}} = 2567 m^3/s$$

The $2567m^{3/}s$ using the Stack Effect calculation plus the additional flow attributed to the density difference in the dryer air is a reasonable crosscheck of the buoyancy calculation which was $3000m^{3/}s$.

The exact calculation of the power output of the wind turbine is complicated and there are a lot of fluid dynamics involved. The Second Law of Thermodynamics (Carnot Efficiency) limits the amount of power out of the wind turbine to the amount of power put into the heat pump minus efficiency loss. An estimate of ~ 50% of the solar energy input of the heat pump should be reasonable for the electrical power output of the wind turbine.

The compression turbine is estimated to take 30% of the gross power. This is only a intuitive estimate and is based on some of the energy put into the compression being recaptured by the power turbine. *More work needs to be done evaluating the performance improvement of compressing the air as it is moving through the heat exchanger and the effects on heat transfer, overall system efficiency and air temperature as it decompresses leaving the heat exchanger.*

Estimated Electrical power output of wind turbine

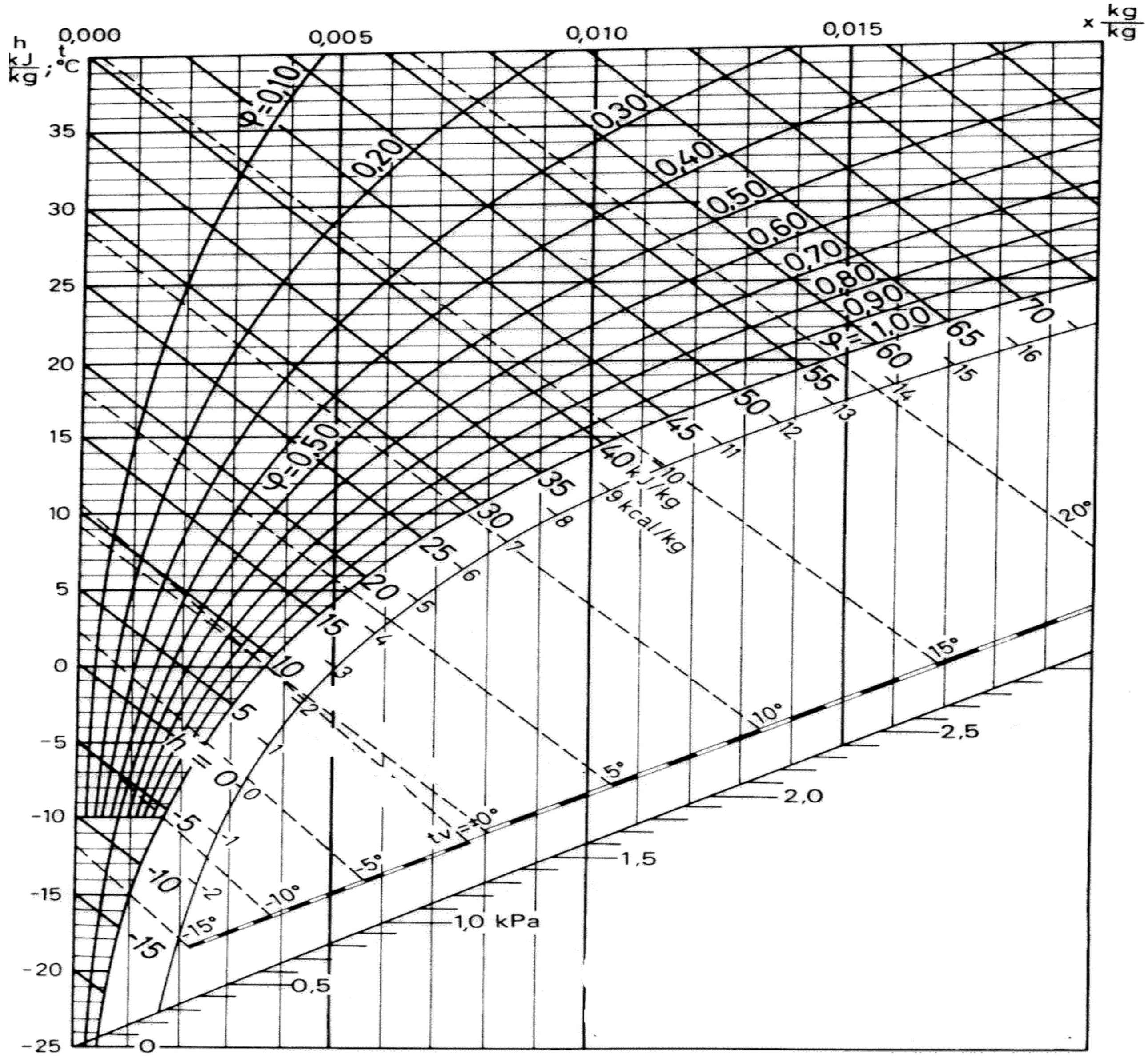
8400 kWh/day Gross – compression turbine =

5880 kWh/day Net

(annual average)

Air Cooling Calculation

Mollier Diagram



Based with an ambient air temperature of 24°C and relative humidity of 70% across the tower height of 100m to lower the temperature of $3000m^3/s$ of air requires removal of 24° X $3000m^3$ X 1.285 kg/m^3 with air having a specific heat of 1.012 J per g per K or 1012 J per Kg per K

1 m^3/s of air at 24^oC and relative humidity 70% (A) is cooled down to 0^oC (B). The surface temperature of the cooling coil is -20^oC (C). The density of air at 24^oC is 1.285 kg/m^3.

Using the Mollier diagram the state of the cooled air (B) is in the intersection between the straight line between (A) and (C) and the 24^{o}C temperature line.

From the Mollier diagram the enthalpy in (A) is 56 kJ/kg, in (B) 8.5 kJ/kg and in (C) -15 kJ/kg.
The Contact Factor can be calculated as:

$$\beta=((56\,kJ/kg)-(8.5\,kJ/kg))/((56\,kJ/kg)-(-15\,kJ/kg))$$
$$=0.669$$

The total heat flow can be calculated as:

$$q=(1\,m3/s)(1.285\,kg/m3)((56\,kJ/kg-8.5\,kJ/kg))$$
$$=61.03(kJ/s,kW)$$

The sensible heat flow can be calculated as:

$$q_s=(1\,m^3/s)(1.285\,kg/m^3)(1.01\,kJ/kg.^{o}C)(24^{o}C-0^{o}C)$$
$$=31.1(kW)$$

According to the Mollier diagram the specific humidity in (A) is 0.096 kg/kg and in (B) 0.001 kg/kg and the latent heat flow can be calculated as:

$$q_s=(1\,m^3/s)(1.285\,kg/m^3)(2{,}502\,kJ/kg)((0.014\,kg/kg)-(0.003\,kg/kg))$$
$$=35.36(kW)$$

At 3000m3/s the heat needed to be removed from the air is:

$$61\mathrm{kJ}/s\times 3000\mathrm{m}^3/s$$
$$=183{,}000\,kJ/s$$
$$=183{,}000\,kW$$
$$=183\,MW$$

To drop the 3000m^3/s of air by 24°C requires approximately 61 kW/m^3

=

183 MW of heat removed from the ambient air moving at 3000 m^3/s
(4.3 GWh$_{(t)}$ per day over 24 hours)

Pressurized Anhydrous Ammonia Requirements

The heat capacity of ammonia vapor is 35.06 J/mol K with molar mass of 17.0304.

$$\boxed{\frac{35.06}{17.0304}=2.059\,J/g\,K}$$

The latent heat of ammonia is 1369 J/g K with a boiling point at atmospheric pressure of -33°C.

To remove 183 MW of heat with liquid ammonia changing state and being warmed to -13°C, the latent heat is approximately 600x the specific heat.

To raise ammonia vapor 20°K it takes 2.059 J X 20 /g = 41.18 Joules per gram.

$$\boxed{\begin{array}{l} g/s\times specific\ heat\times K+g/s*latent\ heat \\ 187{,}000{,}000=Q\times20\times2.059+Q\times1369 \\ \dfrac{187{,}000{,}000}{Q}=41.2+1369 \\ \dfrac{187{,}000{,}000}{Q}=1410.2 \\ Q=132{,}605\,g/s \end{array}}$$

To remove 54 MW of heat from the air:
132 kg/s of liquid ammonia must be evaporated and heated 20°K (to -13°C)

Liquid ammonia density is 681.91 g/l.
The pressurized liquid ammonia input is 132,605g/s at 681.91 g/L

$$\boxed{14000/681.91=194.5\,L/s}$$

194.5 litres/s of liquid ammonia to transfer 187MW of heat from 3000m^2/s of air

Operating 24 hours per day is a total consumption of 194.5 x 24 x 60 x 60 = 16 million litres per day of liquid ammonia.

Ammonia Recovery System

The solar collectors would need to boil enough aqueous ammonia to meet the 194 L/s constant intake during sunlight hours.

Ammonia dissolves in water at 89.9 g/100 ml at 0C.
To absorb the ammonia vapor at the 132 kg/s rate is 132,605 / 899 = 147.5 L/s of cold water.

The absorption of ammonia in water is exothermic (gives off heat), as the ammonia vapor is dissolved the aqueous ammonia solution increases in temperature and absorbs all of the heat energy in the ammonia vapor. The temperature of the water increases and the aqueous ammonia remains at it's vapor point. If there is no increase in pressure, any increase in aqueous ammonia temperature will cause the ammonia to boil.

The aqueous ammonia is pumped to approximately 200 psi pressure prior to entering the heating stage. The pump uses energy but substantially less than a compressor due to the smaller volume. The amount of heat to extract the ammonia is the latent heat of the output ammonia which is 16 million litres or 11 million kg per day plus the heat required to raise the water the same temperature. The aqueous ammonia solution is at approximately 50% and should be at the vapor point at all times.

The heat to boil off the ammonia is:

$$\begin{aligned}&=132{,}605\,g/s\times 1360\,J/g\,K\\&=180\,MJ/s\,,MW\,(12\mathrm{h})\times 2\\&=4.4\,GWh_{(t)}/day\end{aligned}$$

The solar input is over 12h, but it has to recover the ammonia for the 24h period.

Local Solar radiation average 5kWh/m^2. Using a factor of 4 times the latent heat of ammonia:

$$4{,}328{,}227\,kWh/5=866{,}000\,m^2 of\ solar\ panel$$

Solar panel daily input
866,000m^2 x 5KWh/m^2
=
4.4 GWh$_{(t)}$/ day solar thermal input
(during 12 hours daylight)

Water Output

In a high humidity environment the cooling coils produce substantial water. If the heat exchanger is built from stainless steel or glass coated, the water is clean and distilled quality and can be used domestically, for irrigation or for hydrogen electrolysis. The water is also produced high in the tower and may be delivered at a reasonable pressure without pumping energy.

At airflow of 3000m^3/s of 24°C 70% relative humidity air, the tower will extract almost all of the available water vapor.

Saturated Vapor Pressure:

$$\begin{aligned}&E_s=611*10.0^{(7.5*24/(237.7+24))}\\&E_s=2977.4\,Pa\end{aligned}$$

Actual Vapor Pressure:

$$\begin{aligned}&E=(RH\times E_s)/100\\&E=(70\times 29.774)/100\\&E=2084.2\,Pa\end{aligned}$$

Absolute Humidity:

$$\begin{aligned}&D=P/(T\times R_w)\\&Rw=gas\ constant\ for\ water\ vapor=461.5(J/kg\cdot K)\\&D=2084.2/(297.15\times 461)\\&D=0.015\,kg/m^3\end{aligned}$$

Absolute humidity of 24°C 70% RH=0.015kg/m^3

$$3000\text{m}^3/s \times 0.015 = 45\,kg/s$$
$$45\,L/s \text{ of distilled water}$$

The system will produce **45 L/s of distilled water** or almost 4 million litres per day at a height of 100m.

If the water drives a water turbine at the base of the 100m tower, the gross power is:

$$W = \text{flow rate} \cdot g \cdot h$$
$$W = 45 \times 9.8 \times 100\text{m}$$
$$W = 44.1\,kW \text{ gross power}$$

With the efficiency of a water turbine at 50% the net power from the condensed water:

$$W = 44.1\,kW * 0.5$$
$$W = 22\,kW \text{ Net Electrical}$$

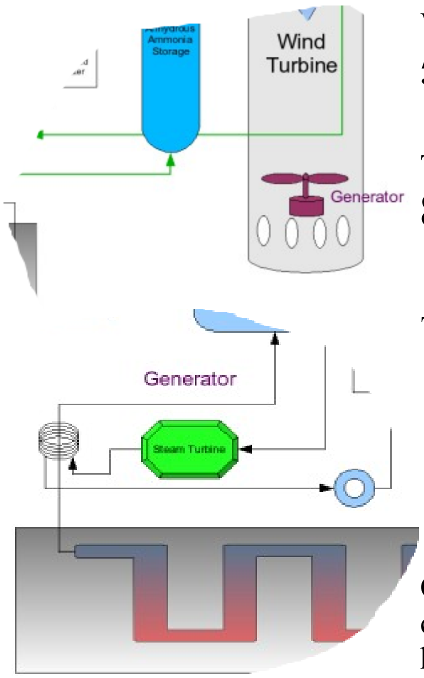
The condensed water capture:

45L/s (4 million L/day) at 100m
which can generate:
22kW (528kWh/day) Net

System Losses

The thermal losses are attempted to be factored into the calculations, but there are losses in the system in pumping the aqueous ammonia, cycling the working fluid in the thermal storage and thermal collection, and the electrical generators. The electrical power output is constant and much higher than any direct solar system with an output of > 40% of the solar thermal input. Heat dissipation is the major engineering issue in this system located in a tropical climate.

Electrical Output



Wind turbine output =

5.8 MWh$_{(e)}$ per day gross output (24x7)

Total gross thermal storage = 184 MW * 24 h + 180MW * 12h * 2 =

8.8 GWh$_{(t)}$ per day of thermal dissipation

The second heat recovery turbine should convert 4% of the the heat going into thermal dissipation into electricity during daylight hours.

184 MW * 24 h + 180MW * 12h =

6,576MWh$_{(t)}$ x 4% = 263 MWh$_{(e)}$ per day gross

(during daylight hours)

One of the major issues in a tropical climate is heat dissipation. Using the condensed water output to cool the steam turbine and providing distilled hot water domestically would be a method of dissipating a portion of the 8.8 GWh$_{(t)}$ of heat. The 4 million L/day of condensed water could be heated to ~ 70°C (cooling the steam turbine) from the 0°C condensation temperature.
Heating the output condensed water would provide clean domestic hot water and dissipate:
45L/s x 4186J/(kg·K) x70 = 13MW$_{(t)}$ x 24h = 316MWh$_{(t)}$ per day

Total System Statistics:

- Wind turbine electrical output: **5.8 MWh$_{(t)}$/day** (constant 24h)
- Heat recovery turbine output: **263 Mwh$_{(e)}$/day** (daylight hours)
- Water turbine electrical output: **528kWh$_{(e)}$/day**
- **Total** electrical output: **270 Mwh$_{(e)}$/day**
- Thermal dissipation: **8.8 Gwh$_{(t)}$/day**
- Water output: **4 million L/day (may be heated to 70°C)**
- 300 m tower
- 866,000 m^2 trough concentrated solar collectors
- large thermal sink (vertical bore-hole 200m deep 1km x 1km= 200,000,000 m^3)
- 8 million litres pressurized anhydrous ammonia storage
- 8 million litres water storage
- 8 million litres aqueous ammonia storage